A.3 Properties of Functions

Saying that a function is *continuous* means it can be drawn without lifting a pencil from the paper (that is, it does not ‘jump’ up or down).

A function, y=f(x), is *smooth* if it has no ‘kinks.’

A function is *monotonically increasing* if *y* goes up every time *x* goes up.

A function is *monotonically decreasing* if *y* goes down every time *x* goes up.

A.4 Inverse Functions

If $y = f(x)$ is monotonic then we can also write $x = g(y)$ where g is also a function. g is the *inverse* function of f (usually written as f^{-1}).

e.g., If $y = f(x) = 2x + 3$, then $x = f^{-1}(x) = \frac{y-3}{2}$ = g(y)

e.g., Demand curves:

q = D(p); p = D^{-1}(q) (Inverse Demand)

The (natural) logarithm or log of x describes a particular function of x which we write $y=f(x) = \ln(x)$. Some logarithm rules:

- $\ln(xy) = \ln(x) + \ln(y)$
- $\ln(x/y) = \ln(x) - \ln(y)$
- $\ln(x^b) = b \ln(x)$
- $\ln(e) = 1$

A.10 Derivatives

Only a linear function has a constant slope. If we "magnify" a smooth curve, then at a particular point it looks like a line. Thus, we can calculate the "instantaneous slope" of a function.

Definition of a Derivative:

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

This gives the instantaneous slope of the curve $y = f(x)$. It is called the derivative of y with respect to x.

Example 1:
Find the instantaneous slope of $y=2x+3$ at $x = 2$.

Example 2:
Find the instantaneous slope of $y=x^2$ at $x = 2$.

Example 3:
Find* the instantaneous slope of $y=\ln x$ at $x = 2$.

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* Note that if f(x) = ln(x), dy/dx = 1/x.

A.11 Second Derivatives

Derivatives are functions – so we can take derivatives of derivatives. These 'second derivatives' tell us how the slope changes.

Example: $y = 100 + 2x - 5x^2$

Then:

$\dfrac{dy}{dx} = 2 - 10x$ -- measures the rate of change

$\dfrac{d^2y}{dx^2} = -10$ -- measures the curvature; "slope of the slope"